April 17, 2024. Period 1

$$\int \frac{\arctan x}{1+x^2}\,dx$$

$u = \arctan x$

$(1+x^2)\,du = \frac{1}{\cancel{1+x^2}}\,dx\ \cancel{(1+x^2)}$

$$\int \frac{u}{\cancel{1+x^2}}\cancel{(1+x^2)}\,du$$

$(1+x^2)\,du = dx$

$$\int u\,du = \frac{1}{2}u^2 + C$$

$$\frac{1}{2}(\arctan x)^2 + C$$

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13. $F(x) = (x-2)^{2/3} + 1$ $\quad [1, 10]$

$F(1) = (1-2)^{2/3} + 1 = (-1)^{2/3} + 1 = \left(\sqrt[3]{-1}\right)^2 + 1 = 2$

$F(10) = (10-2)^{2/3} + 1 = (8^{2/3}) + 1 = \left(\sqrt[3]{8}\right)^2 + 1 = 5$

$F(x) = (x-2)^{2/3} + 1$

Min Max

$F'(x) = 0$ or $\emptyset$

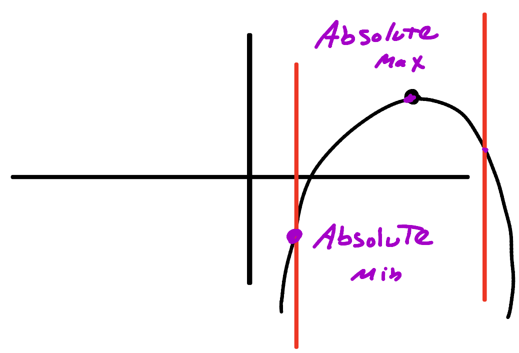
$\sqrt[3]{a\cdot a\cdot a} = a$

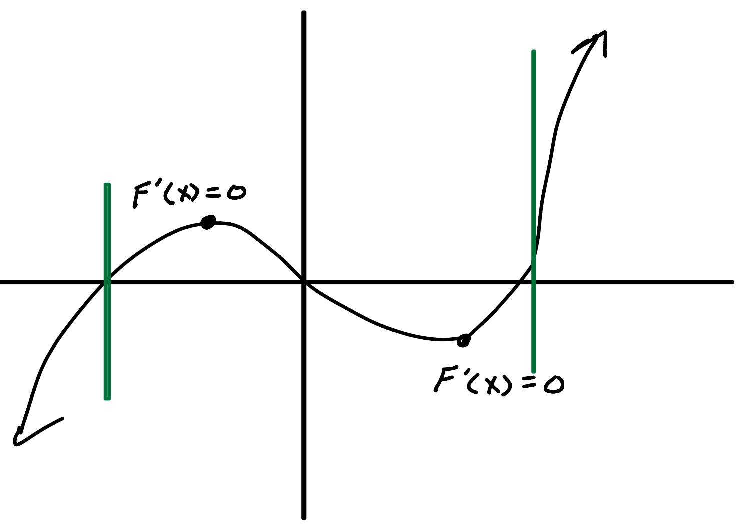
$\sqrt[3]{-1} = \sqrt[3]{-1\cdot -1\cdot -1} = -1$

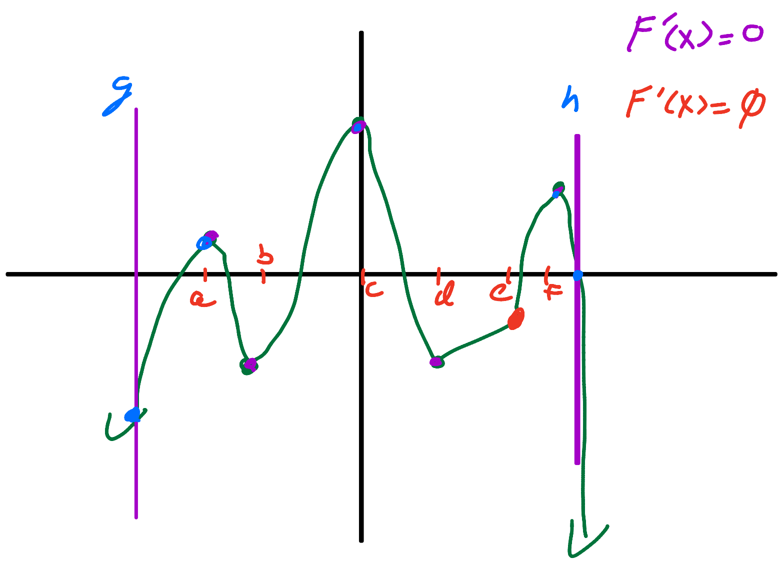
$F'(x) = \frac{2}{3}(x-2)^{-1/3} + 0 = \frac{2}{3(x-2)^{1/3}}$

$x = 2 \quad F'(2) = \emptyset$

$F(2) = (2-2)^{2/3} + 1 = 0 + 1 = 1$







X	F(x)
g	F(g) Ab Min
h	F(h)=0
a	F(a)=R Max
b	
c	F(c) Ab Max
d	
e	
F	F(F) = R Max

20.

$$F(x) = \begin{cases} \dfrac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{For } x \neq 1 \\ k & \text{For } x = 1 \end{cases}$$

$$\lim_{x \to 1} \frac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} = \frac{\sqrt{4} - \sqrt{4}}{1-1} = \frac{0}{0} = \phi$$

$$\lim_{x \to 1} \frac{(x+3)^{\frac{1}{2}} - (3x+1)^{\frac{1}{2}}}{x-1}$$

$$\lim_{x \to 1} \frac{\frac{1}{2}(x+3)^{-\frac{1}{2}} - \frac{1}{2}(3x+1)^{-\frac{1}{2}} \cdot 3}{1} = \frac{\frac{1}{2\sqrt{x+3}} - \frac{3}{2\sqrt{3x+1}}}{1}$$

$$\frac{\frac{1}{2\sqrt{4}} - \frac{3}{2\sqrt{4}}}{1} = \frac{1}{4} - \frac{3}{4} = \frac{-1}{2}$$

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21 $\quad F(x) = \dfrac{x}{2x-3}$ $\qquad (1, F(1))$

$$F(1) = \frac{1}{2(1)-3} = \frac{1}{-1} = -1 \qquad \text{Point } (1,-1)$$

$$F'(x) = \frac{1(2x-3) - x(2)}{(2x-3)^2} = \frac{(2(1)-3) - 1 \cdot 2}{(2-3)^2} = \frac{-1-2}{1} = \underline{-3}$$

Slope of Tangent Line

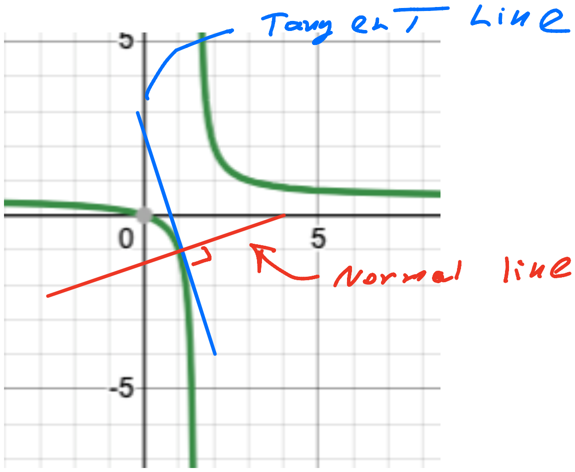
Normal $m = +\frac{1}{3}$

Point $(1,-1)$

$$y = \frac{1}{3}x + b$$

$$-1 = \frac{1}{3}(1) + b$$

$$-\frac{4}{3} = b \qquad 3 \cdot y = \left(\frac{1}{3}x - \frac{4}{3}\right) \cdot 3$$



22.

$F(x) = x \cdot \ln x$

$F'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$0 = \ln x + 1$

$-1 \qquad -1$

$-1 = \ln x$

$x = \frac{1}{e}$

$F(\frac{1}{e}) = \frac{1}{e} \cdot \ln \frac{1}{e}$

$= \frac{1}{e} \cdot -1$

$-\frac{1}{e}$ ← what is min

$\ln \frac{1}{e}$

$\ln e^{-1}$

$-1 \ln e$

$-1 \cdot 1$

-1

$\ln_e x = -1$

$e^{-1} = x$

$\frac{1}{e} = x$ ← When min Happens

$\ln e = 1$

$\ln e^{-1}$

$-1 \ln e$

$-1 \cdot 1$

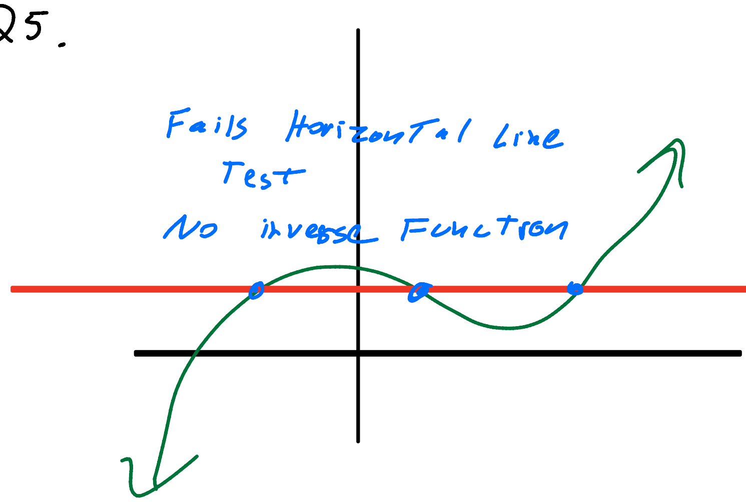
-1

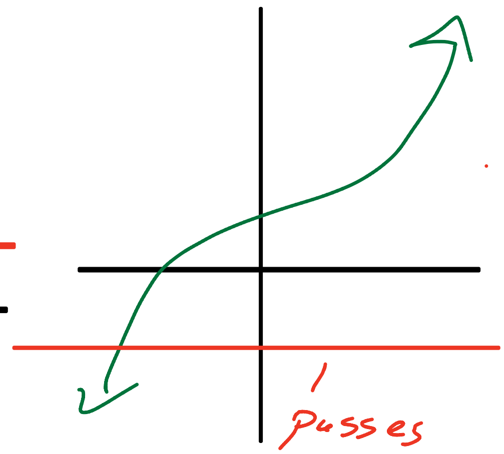
$\log_6 x = -5$

$6^{-5} = x$

$\frac{1}{6^5} = x$

25.





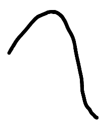
Passes Horizontal Line Test

Has inverse Function

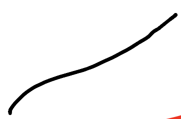
$g'(x) < 1$

$g'(x) = .5$

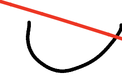
$g'(x) = -5$



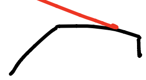
$g'(x) > 1$ always increasing



$g''(x) > 0$ concave UP



$g''(x) < 0$ concave DN



27

$$G(x) = \int_0^{2x} \cos(T^2)\,dT \qquad T = 2x \quad dT = 2dx$$

$$G'(x) = \cos(2x)^2 \cdot 2$$

$$G'(\sqrt{\pi}) = (\cos 4(\sqrt{\pi})^2) \cdot 2$$

$$(\cos 4\pi) \cdot 2$$

$$1 \cdot 2 = 2$$

28.

$$\frac{d\,pro}{dT} = 6\,\text{gal/min} \quad \text{in}$$

$$\frac{1}{\sqrt{T+1}}\,\text{g/m} \quad \text{OUT}$$

$T=0 \quad 20\,\text{gal}$

in $\quad 8$ min $\quad 6 \cdot 8 = 48$ gallons

out $\quad \int_0^8 \frac{1}{\sqrt{T+1}}\,dT$

$$u = T+1$$
$$du = dT$$

$$\int \frac{1}{\sqrt{u}}\,du = \int u^{-\frac{1}{2}}\,du = \frac{2}{1} \cdot u^{-\frac{1}{2}+1=\frac{1}{2}} + C$$

$$2\sqrt{T+1}\,\Big|_0^8$$

$$2\sqrt{8+1} - 2\sqrt{0+1}$$

$$2\sqrt{9} - 2\sqrt{1}$$

$$2 \cdot 3 - 2 \cdot 1$$

$$6 - 2 = 4$$

$$\int_0^8 \left(6 - \frac{1}{\sqrt{T+1}}\right)dT$$

$$= 44$$

$$48 - 4$$

$$44$$

$$+20$$

$$64\,g$$

29.

$a = 4e^{2T}$

$T=0 \quad S(0)=2$

$V(0)=-2$

$S(\frac{1}{2})=?$

$\int a(T) = V(T)$

$\int 4e^{2T}\,dT = \int 4e^{u}\cdot\frac{du}{2} = \frac{4}{2}\int e^{u}\,du = 2e^{u}+C$

$u=2T$

$du=2dT$

$\frac{du}{2}=dT$

$V(T) = 2e^{2T}+C$

$V(0) = 2\cdot e^{0}+C = -2$

$2+C=-2$

$C=-4$

$V(T) = 2e^{2T}-4$

$S(T) = \int (2e^{2T}-4)\,dT$

$S(T) = e^{2T}-4T+C$

$S(0) = e^{0}-4(0)+C=2 \qquad C=1$

$S(T) = e^{2T}-4T+1$

$S(\frac{1}{2}) = e^{1}-2+1 = e-1$

34.

$F(x) = \frac{(x-1)^2}{2x^2-5x+3} = \frac{(x-1)^2}{\underbrace{2x^2-2x}_{2x(x-1)}\underbrace{-3x+3}_{-3(x-1)}} = \frac{(x-1)^2}{(x-1)(2x-3)}$

$F(+1) = \emptyset = \frac{0}{0}$

graph $y=\frac{(x-1)}{2x-3}$

Hole at $x=1$

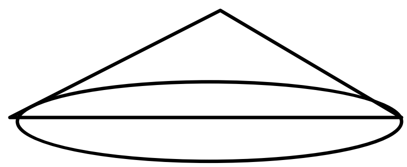
Vertical asy $2x-3=0 \quad x=\frac{3}{2}$

Horizontal $\lim_{x\to\pm\infty} \frac{(x-1)}{2x-3} = \frac{1}{2}$

36.

$r = 3h \Rightarrow h = \frac{1}{3}r$

$V = \frac{1}{3}\pi r^2 \cdot h = \frac{1}{3}\pi r^2 \cdot \frac{1}{3} r$



$\frac{dr}{dT}$

$V = \frac{1}{9}\pi r^3$

$\frac{dV}{dT} = \frac{\pi}{9} \cdot 3r^2 \cdot \frac{dr}{dT}$

V at 6 min

$V = 4\pi$

r at 6 min

$\cancel{\frac{9}{\pi}} \cdot 4\cancel{\pi} = \frac{1}{\cancel{9}}\cancel{\pi} r^3 \cancel{\cdot \frac{9}{\pi}}$

$36 = r^3$

$\sqrt[3]{36} = r$

Slope at $T = 6$ of graph of $V(T) \Rightarrow \pi$

$V(5) = 3\frac{1}{2}\pi = \frac{7}{2}\pi$

$V(7) = 5\pi$

$\frac{5\pi - 3\frac{1}{2}\pi}{2} = \frac{1\frac{1}{2}\pi}{2} = \frac{\frac{3\pi}{2}}{2}$

$V'(6) = \frac{3}{4}\pi$

$\frac{3}{r^2\pi} \cdot \frac{dV}{dT} = \cancel{\frac{\pi}{3}} r^2 \frac{dr}{dT} \cdot \cancel{\frac{3}{r^2\pi}}$

$\frac{3}{r^2 \cdot \cancel{\pi}} \cdot \cancel{\pi} = \frac{dr}{dT}$

38. $5x^3 + 40 = \int_a^x F(T)\,dT$ The value of a is?

$5x^3 + 40 = \int_a^x 15T^2\,dT$

$F(x) = 15x^2$ $F(T) = 15T^2$

$\int 15x^2\,dx = 5x^3 + C \Big[_a^x = 5x^3 + 40$

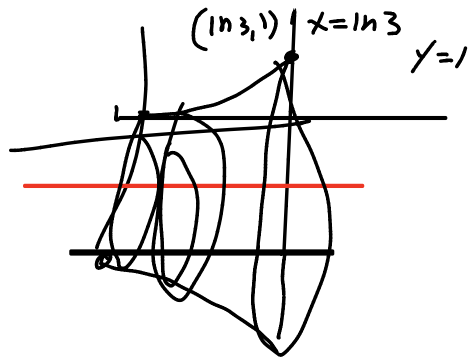
$5x^3 + 40 = 5x^3 + c - [5a^3 + c]$

$5x^3 + c - [5a^3 + c] = 5x^3 + 40$

$5x^3 + 40 = 5x^3 - 5a^3$

$40 = -5a^3$ $8 = -a^3$ $a^3 = -8$ $a = -2$

39.



$r = 2$

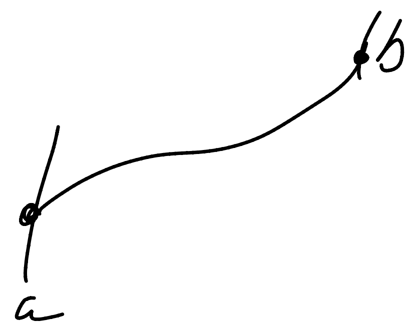
$R = e^{x/2} + 1$

$$\int_0^{\ln 3} \pi\left((e^{x/2}+1)^2 - (2)^2\right)dx$$

$$F'(c) = \frac{F(b) - F(a)}{b-a} = MVT \quad \text{True}$$

$[a, b]$

$F'(c) = 0$



$\frac{dy}{dx} = y^2$ $\quad y(-1) = 1$

$\cancel{y\cdot} -\frac{1}{\cancel{y}} = x \cdot y$

$$\int \frac{1}{y^2}dy = \int dx$$

$-\frac{1}{y} = x + C$

$-\frac{1}{1} = -1 + C$

$0 = C$

$\frac{-1}{x} = \frac{\cancel{x}y}{\cancel{x}}$